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Distribution systems design with role dependent objectives

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ABSTRACT

In this paper we introduce flexible models for capacitated discrete location problems. We describe three different points of view of a location problem in a logistics system. Different mathematical programming formulations are presented, illustrated by examples and compared using a battery of test problems. Extensive computational tests are done showing the potentials and limits of this kind of resolution approach.

1. Introduction

An essential part in designing and planning logistics systems is the identification of the main cost drivers in a certain planning task to perform an optimization with a special focus on these cost drivers. From a client point of view one is (for example) interested in identifying the most expensive customers (with respect to delivery costs). This situation occurs in problems where customers travel to receive a service and the main goal is to minimize the transportation cost from the customers to the servicing facilities which are supported by the customers. Focusing instead on the suppliers one may want to put an emphasis on the most costly suppliers. This case happens, for instance, when establishing franchises of a parcel company to cover door-to-door deliveries in a given region. In this situation each franchise supports the distribution cost and the company pays to the franchises a fixed amount of money per parcel delivered in the region. The goal of the company is to pay as less as possible to their franchises but still ensure enough benefit to the franchises (otherwise, no subcontractor would buy the right for the franchise). Therefore, the company has to find places for these franchises that produce the minimum distribution cost. A 3PL (3rd party logistics provider) however would set the focus on the costs of the individual transportation links without adding up costs on the customer or supplier side. This is the case of hazardous material (haz-mat) transportation activities. In these situations the hazardous materials are shipped from the producing centers to landfills and the overall goal is to shorten as much as possible each transportation link to minimize environmental exposure to risk. Moreover, classical objective functions, apart from not representing the different points of view in the logistic problem, simply sum up all costs and it is impossible to formulate an objective function which only takes care of the say 10 highest terms in the overall cost. In this paper we are proposing a first attempt to close this gap in the context of supply chain design with a focus on the location side.

Discrete location problems typically involve a finite set of sites at which facilities can be located, and a finite set of clients, whose demands for service or goods have to be fulfilled by the facilities. The most simple and well studied discrete location problems are the discrete p-median problem and the Uncapacitated (and Capacitated) Facility Location Problem, see Aardal (1998) and Neame et al. (2000). Evidently, many extensions of these basic location problems have been developed. The extensions range from capacity restrictions, over multi-echelon structures to time dynamic models. Recently, several articles have been published addressing strategic supply chain decisions in the context of location problems, see Hinojosa et al. (2000, 2008), Melo et al. (2009) and references therein. This development led to a highly flexible and general framework of location models in terms of side constraints.

Another important aspect of a location model is the correct choice of the objective function and in most classical location models the objective function is the main differentiator. Therefore, a great variety of objective functions have been considered. The median objective is to minimize the sum of the costs of fulfilling all demand requests from the clients. The center objective is to minimize

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over all clients the maximum costs of fulfilling the demand of a client from the sites chosen. The centdian objective is a convex combination of the median and center objectives; it aims to keep both the average costs behavior as well as the highest costs in balance. Despite the fact that all three objectives (and some more) are frequently encountered in the literature (Drezner, 1995; Drezner and Hamacher, 2002), not much has been done in the direction of a unified framework for handling all these objectives.

The increasing need for discrete location models in strategic supply chain planning (see, e.g., Mirchandani and Francis (1990), Drezner (1995), Kalcsics et al. (2000), Drezner and Hamacher (2002), Melo et al. (2006, 2009) and references therein) has made it necessary to develop new and flexible location models. To that end, Nickel (2001) introduced a new type of objective function which generalizes the most popular objective functions mentioned above. This objective function applies a penalty to the costs of fulfilling the demand of a client which is dependent on the position of that cost relative to the costs of supplying other customers. For example, a different penalty might be applied if the costs of supplying the client were the 5th-most expensive costs rather that the 2nd-most expensive. It is even possible to neglect some customers by assigning a zero penalty. This adds a "sorting"-problem to the underlying facility location problem, making formulation and solution much more challenging.

For planar and network location problems, this generalized model was studied in Puerto and Fernández (1995, 2000), Nickel and Puerto (1999), Francis et al. (2000) and Rodríguez-Chía et al. (2000). In Nickel et al. (2005) the multiobjective problem was studied as well. The research in this area even led to a recent monograph, see Nickel and Puerto (2005). The model that deals with the discrete location problem, namely the discrete ordered median problem (DOMP), has also been analyzed. Starting with a nonlinear formulation, several linearizations have been developed. Structural results as well as a specially tailored branch and bound procedure are presented in Boland et al. (2006). By using a different approach, following the principal idea of Elloumi et al. (2004), an improved branch and cut algorithm was developed in Marín et al. (2006, 2008). This way, it is now possible to optimally solve DOMPs with more than 100 clients in reasonable time. Moreover, a VNS-heuristic and an evolutionary approach can be found in Domínguez-Marín et al. (2005) and different genetic algorithms in Stanimirovic et al. (2007).

In the analysis followed in the paper the usage of these objective functions is instrumental and thus, for the sake of completeness, we start by introducing the basic discrete ordered median problem (DOMP).

Let A denote a given set of M client sites and identify these with integers $1,\dots,M$. In the following we assume without loss of generality that the set of candidate sites for new facilities is identical to the set of clients A. Let $c_{ij} \geqslant 0$ denote the costs of satisfying all the demand of client i from a facility located at site j. Let $N \leqslant M$ be the number of facilities to be located and $X \subseteq A$ with $|X| = N$ denotes a solution. We assume that each client i will be supplied from a site $j \in X$ such that $c_{ij} = c_i(X) := \min_{k \in X} c_{ik}$.

We define σ_X to be a permutation on $\{1,\dots,M\}$ for which the inequalities

$$c_{\sigma_X(1)}(X) \leqslant c_{\sigma_X(2)}(X) \leqslant \cdots \leqslant c_{\sigma_X(M)}(X)$$

hold. Let $\lambda = (\lambda_1,\dots,\lambda_M)$ with $\lambda_i \geqslant 0, i = 1,\dots,M$. The discrete ordered median problem (DOMP) is defined as

$$\min_{X \subseteq A\,,\ |X|=N} \sum_{i=1}^{M} \lambda_i c_{\sigma_X(i)}(X)\ .$$

For different choices of λ we obtain different types of objective functions. Clearly, classical location problems, like the N-median, N-center, N-α-centdian, etc., can easily be modeled under this common pattern. Moreover, new meaningful objective functions are easily derived. Observe that the DOMP is NP-hard, as it is a generalization of the N-median problem (Kariv and Hakimi, 1979). Apart from the modeling aspect of the λ-weights, they also have neat economic interpretations as correction factors for the distance from the users to the servicing facilities. Another usage is for modeling public services that are rather cheap for closest users and are also subsidized for remote ones. In this case, $\wedge$-shaped (increasing–decreasing) correction factors would be suitable to deal with this kind of situation, e.g., (0,2,4,6,5,3,1).

In this paper we extend the model of the basic DOMP to cope with actual requirements from logistics as described above. We present models taking capacities into account. Moreover, we introduce three different points of view on the problem that depend on the member of the logistics network that is the driving force of the planning process: the client, the supplier, and the logistics provider point of view. To illustrate the different views consider the following example.

Example 1.1. Consider a warehouse distribution problem with four possible locations for new warehouses and six wholesalers which have to be served. The demand of the wholesalers is given as: (10,12,11,15,13,14) and the capacities of the possible warehouse locations are: (39,38,37,38). The wholesaler sites are indexed $\{1,\dots,6\}$ and the warehouses $\{1,\dots,4\}$. Moreover, we want to build exactly two new warehouses.

Finally, we are given a transportation cost matrix C associated to the set of wholesalers and possible warehouse locations, where c_{ij} denotes the unit transportation cost between warehouse i and wholesaler j:

$$C = \begin{pmatrix} 12.4 & 57.9 & 19.1 & 21.0 & 17.5 & 20.0 \\ 11.0 & 35.9 & 13.6 & 23.0 & 27.5 & 22.0 \\ 15.0 & 5.9 & 19.1 & 17.0 & 7.5 & 24.0 \\ 11.0 & 7.9 & 21.1 & 27.0 & 8.5 & 26.0 \end{pmatrix}$$

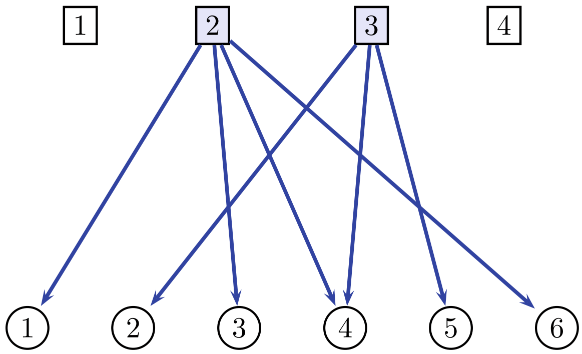
Note that for the ease of understanding, just for this example, we do not assume that the set of clients, i.e., wholesalers, coincides with the set of potential locations for new warehouses.

Classical location problems usually address the client driven model. Namely, a set of facilities should be located such that the average or maximal costs for a client to obtain service at or from a new facility are as small as possible. For example, the average or maximal shipping costs to be paid by the clients for obtaining deliveries from a warehouse should be minimized. Therefore, we denote these models as client cost models.

Example 1.1. (cont.) If we want to minimize the average shipping costs from the warehouses to the wholesalers over all wholesalers, it is optimal to locate the two warehouses at the sites 2 and 3 with average shipping costs of 168.15, see also Fig. 1.

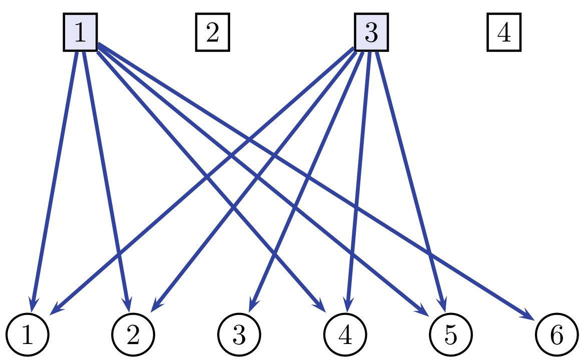
If, however, each wholesaler has to bear the costs to obtain his products by himself, a more appropriate objective would be to minimize the maximal shipping costs for each wholesaler when determining new warehouse locations. In that way we can take into account aspects of fairness in the decision making process in the sense that the costs should be balanced over all wholesalers and nobody has to burden exceptionally high transportation costs reducing his profit margin unduly. For this situation it is optimal to locate the two warehouses at the sites 1 and 3 yielding maximal shipping costs of 280, see also Fig. 2.

Recently, Zhou et al. (2002) introduced a different point of view. They discuss an application where the product flows within the supply chain network were required to be as equitable as possible



Allocation	Supplied Amount
$2 \longrightarrow 1$	10
$3 \longrightarrow 2$	12
$2 \longrightarrow 3$	11
$2 \longrightarrow 4$	3
$3 \longrightarrow 4$	12
$3 \longrightarrow 5$	13
$2 \longrightarrow 6$	14

Fig. 1. Location and allocation pattern for the client point of view minimizing the average shipping costs.



Allocation	Supplied Amount
$1 \longrightarrow 1$	2.47
$3 \longrightarrow 1$	7.53
$1 \longrightarrow 2$	4.02
$3 \longrightarrow 2$	7.98
$3 \longrightarrow 3$	11
$1 \longrightarrow 4$	6.25
$3 \longrightarrow 4$	8.75
$1 \longrightarrow 5$	11.26
$3 \longrightarrow 5$	1.74
$1 \longrightarrow 6$	14

Fig. 2. Location and allocation pattern for the client point of view minimizing maximal shipping costs per wholesaler.

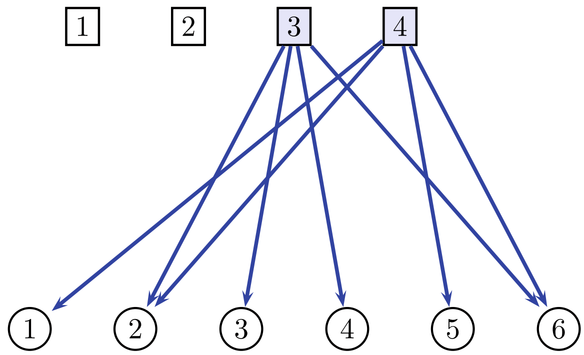
in the sense that each distribution center is given the same amount of workload. Their rationale is, that the balanced allocation of customers to distribution centers increases the chances of minimizing stock-outs and late deliveries, while maximizing the order fill rate and utilization rate of distribution centers. They solve the problem by minimizing over all distribution centers the maximal total shipping costs of a distribution center. Hence, the focus is on the distribution centers, i.e., the supply facilities, and not on the clients. Consequently, we denote these models as supplier cost models.

Note, however, that the two models are identical if we are just looking at the overall costs, i.e., we use a median type objective. Only when considering other objective functions, like the minimax objective as in Zhou et al. (2002), we obtain different results, as we will see later on.

Example 1.1. (cont.) Following the model in Zhou we try to balance the workload in the warehouses by minimizing the maximal costs per warehouse. For this situation it is optimal to locate the two warehouses at the sites 3 and 4 with maximal costs per warehouse of 561.2, see also Fig. 3.

The last model discusses the case in which a 3rd party logistics provider is involved as well. In contrast to the previous two models, the latter does not have an accumulated point of view but looks instead at each transport relation separately, see also Newman et al. (2005).

A motivation to consider this point of view is, for example, founded in the system-inherent Bull-whip effect of Supply Chains. One possibility to reduce this effect, and consequently the overall costs, is to reduce the order lead times, see, e.g., Simchi-Levi et al. (2003). Order lead times, in turn, significantly depend on transportation times. Therefore, to reduce the Bull-whip effect, one should try to minimize the maximal shipping time of a delivery, i.e., of a single transportation relation. We call this model the



Allocation	Supplied Amount
$4 \longrightarrow 1$	10
$3 \longrightarrow 2$	9.28
$4 \longrightarrow 2$	2.72
$3 \longrightarrow 3$	11
$3 \longrightarrow 4$	15
$4 \longrightarrow 5$	13
$3 \longrightarrow 6$	1.72
$4 \longrightarrow 6$	12.28

Fig. 3. Location and allocation pattern for the supplier point of view.

logistics provider cost model. Note, that it is identical to the previous two models if we use a median type objective. Moreover, it coincides with the client cost model if the problem is uncapacitated.

Example 1.1. (cont.) Following the above motivation, assume for the moment that the entries in the cost matrix C denote transportation times instead of transportation costs. As considering just the largest transportation time might be too restrictive, we want to find new locations for the warehouses such that the sum of the 3 largest transportation times are minimal.

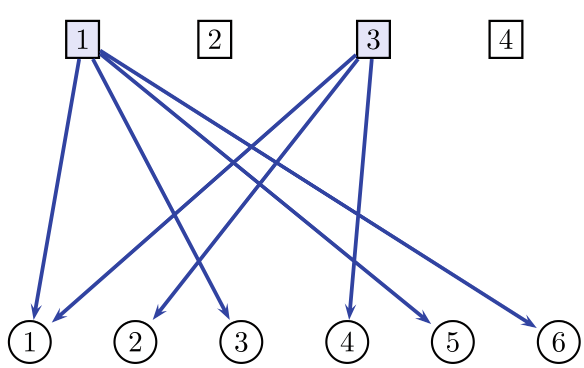
For this situation it is optimal to locate the two warehouses at the sites 1 and 3 with objective value 56.6, see also Fig. 4. The largest lead time is thereby 20.

The above example shows that each point of view gives rise to very different solutions that represent the different interests of the parties in a logistics problem. We are not aware of any similar analysis focused on supply chain design previously published in the literature.

There are other applications where different points of view are already employed, although not explicitly stated as such. For example in scheduling problems, finding a schedule that minimizes the total or maximal due date violation(s) can be interpreted as a client point of view (we want to meet the customers requirements "as good as possible"). In contrast, minimizing the total makespan sets the focus on the supplier, i.e., producer (his machines should be utilized in the best possible way), see Pinedo (2008). Also in the control of queues it is well-known that using different queue disciplines lead to different ways to undertake the problem. For instance applying head of line (HOL) discipline minimizes the server's average cost per second which reflects the server point of view, whereas a first come first served (FCFS) discipline clearly focuses on the client point of view, see Kleinrock (1976).

In this paper we will develop mathematical programming models that capture the different characteristics in the above three points of view of a logistics problem. These three points of view will be represented within a flexible family of objective functions: the *ordered median functions*. We present several formulations and compare their performance by means of computational experiments on randomly generated test instances.

The rest of the paper is organized as follows: In Section 2 we provide mathematical formulations for the different points of view.



Allocation	Supplied Amount
$1 \longrightarrow 1$	1
$3 \longrightarrow 1$	9
$3 \longrightarrow 2$	12
$1 \longrightarrow 3$	11
$3 \longrightarrow 4$	15
$1 \longrightarrow 5$	13
$1 \longrightarrow 6$	14

Fig. 4. Location and allocation pattern for the logistics provider point of view.

Section 3 studies alternative formulations for the capacitated discrete ordered median problem under the hypothesis of λ-weights given in non-decreasing order. In Section 4, we present a computational analysis to determine the limits of solving the problem with different formulations using standard MIP solvers. The paper ends with some conclusions.

2. Modeling the different points of view of a logistics problem

As described in the introduction, the goal of this paper is to develop flexible mathematical programming models that capture three different points of view that appear in a logistics problem, and that are capable to incorporate very different objective functions and constraints. In our approach to model the different points of view of a logistics problem, we borrow some flexible formulations already available in the literature. The necessity for a unified way of representing very different objective functions that are associated to the different points of view, leads us to use the *ordered median function* (see Nickel and Puerto, 2005).

In the following we address this modeling phase by presenting formulations of the capacitated discrete ordered median problem (CDOMP) for the three different points of view. In the first subsection, which is devoted to the client point of view, we compare three alternative formulations based on different sets of variables (with 3, 2 and 1 indexes, respectively) and discuss their efficiency. The second subsection introduces the supplier point of view formulation, whereas the third presents the formulation of the problem under the logistics provider point of view.

We will denote a_k the demand of client k, for all $k = 1, \dots, M$ and b_j the capacity of a supplier located at site j, for all $j = 1, \dots, M$. Recall that we want to select N out of the M candidates sites as locations for new facilities.

2.1. Modeling from a client point of view

As mentioned above, we embed our models into the family of ordered median problems. To this end, we consider the capacitated discrete ordered median problem from a client point of view (CDOMP_{CV}), also denoted client cost model. This model looks for a given number of supply facilities with enough capacity to cover the entire demand of the clients. The goal is to minimize the weighted sum of the costs to cover the overall demand of the clients (client costs), where the weights are correction factors that depend on the ordered sequence of client costs. In order to formulate this model we use the following set of variables:

s_{ijk} = proportion of demand a_k, covered by supplier j when the total transportation costs to cover the demand of client k are in the ith position of the client cost vector.

$$y_j = \begin{cases} 1 & \text{if a new supplier is located at site } j, \\ 0 & \text{otherwise,} \end{cases}$$

$$z_{ik} = \begin{cases} 1 & \text{if the overall transportation costs to cover client } k \text{ are at position } i, \\ 0 & \text{otherwise,} \end{cases}$$

for $i, j, k = 1, \dots, M$. Among the above variables, the y_j are standard in location models, see Drezner (1995), while the remainders are specific to the discrete ordered models. The mathematical model, denoted ($CDOMP_{CV}$), is now as follows:

$$\min \quad \sum_{i=1}^{M} \lambda_i \sum_{j=1}^{M} \sum_{k=1}^{M} s_{ijk}\, a_k\, c_{jk} \tag{1}$$

$$\text{s.t.} \quad \sum_{i=1}^{M} \sum_{j=1}^{M} s_{ijk} = 1 \quad \forall k = 1, \dots, M \tag{2}$$

$$\sum_{j=1}^{M} \sum_{k=1}^{M} s_{ijk} = 1 \quad \forall i = 1, \dots, M \tag{3}$$

$$\sum_{j=1}^{M} s_{ijk} = z_{ik} \quad \forall i, k = 1, \dots, M \tag{4}$$

$$\sum_{j=1}^{M} \sum_{k=1}^{M} s_{ijk}\, a_k\, c_{jk} \leqslant \sum_{j=1}^{M} \sum_{k=1}^{M} s_{i+1,jk}\, a_k c_{jk} \quad \forall i = 1, \dots, M-1 \tag{5}$$

$$\sum_{i=1}^{M} \sum_{k=1}^{M} s_{ijk}\, a_k \leqslant b_j y_j \; \forall \quad j = 1, \dots, M \tag{6}$$

$$\sum_{j=1}^{M} y_j = N \tag{7}$$

$$y_j \in \{0, 1\} \quad \forall j = 1, \dots, M$$
$$s_{ijk} \geqslant 0 \quad \forall i, j, k = 1, \dots, M$$
$$z_{ik} \in \{0, 1\} \quad \forall i, k = 1, \dots, M$$

The objective function (1) is the weighted ordered sum of the total client costs (the client cost for a client in position i is $\sum_{j=1}^{M} \sum_{k=1}^{M} s_{ijk} a_k c_{jk}$). Constraints (2) guarantee that the demand of all clients is covered. Constraints (3) and (4) ensure that total transportation costs to cover a client may be allocated to only one position in the ordered vector of total client costs. Constraints (5) guarantee the non-decreasing order of the entries of the client cost vector. Constraints (6) ensure that the total amount supplied by each server does not exceed its capacity. By constraint (7), N new supply facilities are built.

Note that the quality of the lower bound provided by the linear relaxation of the above formulation is usually very bad. Indeed, in the relaxed problem it is possible for each client to satisfy its demand using self-service by partially opening all facilities, that is, each client is served by itself whenever it has enough capacity. Therefore, for the problems with free self-service, the lower bound obtained by the linear relaxation could be zero. This property applies to all the remaining models and therefore forces to develop new types of lower bounds to be used in algorithmic approaches see Nickel and Puerto (2005).

2.1.1. Alternative formulations for the CDOMP_{CV}

Replacing the term $\sum_{j=1}^{M} s_{ijk}\, a_k c_{jk}$ by a single, two-index variable t_{ik} gives rise to a new formulation of the problem. This model uses the following additional sets of variables:

x_{jk} = proportion of demand a_k covered by the supplier j
t_{ik} = total transportation costs to cover client k at position i

for $i, j, k = 1, \dots, M$. The formulation of the problem, denoted ($CDOMP^2_{CV}$), is then as follows:

$$\min \quad \sum_{i=1}^{M} \lambda_i \sum_{k=1}^{M} t_{ik}$$

$$\text{s.t.} \quad \sum_{j=1}^{M} x_{jk} = 1 \quad \forall k = 1, \dots, M \tag{8}$$

$$\sum_{i=1}^{M} z_{ik} = 1 \quad \forall k = 1, \dots, M \tag{9}$$

$$\sum_{k=1}^{M} z_{ik} \leqslant 1 \quad \forall i = 1, \dots, M \tag{10}$$

$$\sum_{k=1}^{M} t_{ik} \leqslant \sum_{k=1}^{M} t_{i+1,k} \quad \forall i = 1, \dots, M-1 \tag{11}$$

$$\sum_{k=1}^{M} a_k x_{jk} \leqslant b_j y_j \quad \forall j = 1,\dots,M \tag{12}$$

$$t_{ik} \geqslant \sum_{j=1}^{M} a_k c_{jk} x_{jk} - \sum_{j=1}^{M} a_k c_{jk}(1 - z_{ik}) \quad \forall i,k = 1,\dots,M \tag{13}$$

$$\sum_{j=1}^{M} y_j = N$$

$$z_{ik} \in \{0,1\} \quad \forall i,j,k = 1,\dots,M$$

$$y_j \in \{0,1\} \quad \forall i,j,k = 1,\dots,M$$

$$0 \leqslant x_{ij} \leqslant 1 \quad \forall i,j = 1,\dots,M$$

$$t_{ik} \geqslant 0 \quad \forall i,k = 1,\dots,M$$

Constraints (8)–(10) again ensure that the demand of all clients is met and that the total transportation costs to cover a client may be allocated to only one position in the ordered vector of total client costs.

Constraints (11) guarantee the non-decreasing order of the entries of the client cost vector. Constraints (12) ensure that the total amount supplied by each server does not exceed its capacity. Finally, (13) guarantees that the t_{ik} variables are correctly modeled.

Note that we obtain an alternative formulation by replacing constraints (13) with the following two sets of constraints:

$$\sum_{i=1}^{M} t_{ik} = \sum_{j=1}^{M} x_{jk} a_k c_{jk}, \quad \forall k = 1,\dots,M,$$

and

$$t_{ik} \leqslant z_{ik} \max_{j=1,\dots,M} a_k c_{jk}, \quad \forall i,k = 1,\dots,M.$$

Yet another formulation can be obtained by further aggregating the variables t_{ik}. Let

$t_i =$ the transportation costs in the ith position, $i = 1,\dots,M$.

By doing so, we obtain the following model, denoted $(CDOMP^1_{CV})$:

$$\min \quad \sum_{i=1}^{M} \lambda_i t_i$$

$$\text{s.t.} \quad \sum_{j=1}^{M} x_{jk} = 1 \quad \forall k = 1,\dots,M$$

$$\sum_{i=1}^{M} z_{ik} = 1 \quad \forall k = 1,\dots,M$$

$$\sum_{k=1}^{M} z_{ik} \leqslant 1 \quad \forall i = 1,\dots,M$$

$$t_i \leqslant t_{i+1} \quad \forall i = 1,\dots,M-1 \tag{14}$$

$$\sum_{k=1}^{M} a_k x_{jk} \leqslant b_j y_j \quad \forall j = 1,\dots,M$$

$$t_i \geqslant \sum_{j=1}^{M} a_k c_{jk} x_{jk} - \sum_{j=1}^{M} a_k c_{jk}(1 - z_{ik}) \quad \forall i,k = 1,\dots,M \tag{15}$$

$$\sum_{j=1}^{M} y_j = N$$

$$y_j \in \{0,1\} \quad \forall i,j,k = 1,\dots,M$$

$$z_{ik} \in \{0,1\} \quad \forall i,j,k = 1,\dots,M$$

$$0 \leqslant x_{ij} \leqslant 1 \quad \forall i,j = 1,\dots,M$$

$$t_i \geqslant 0 \quad \forall i = 1,\dots,M$$

Except for Constraints (14) and (15), which have the same meaning as (11) and (13), all other constraints are identical to the previous model. Note that, again, an alternative formulation can be obtained from the previous one by replacing constraints (15) with:

$$\sum_{i=1}^{M} t_i = \sum_{k=1}^{M} \sum_{j=1}^{M} x_{jk} a_k c_{jk},$$

and

$$t_i \leqslant \sum_{k=1}^{M} z_{ik} \max_{j=1,\dots,M} a_k c_{jk}, \quad \forall i = 1,\dots,M.$$

In the following, we denote the linear programming relaxation of $\left(CDOMP^1_{CV}\right)$ and $\left(CDOMP^2_{CV}\right)$ by $\left(LP - CDOMP^1_{CV}\right)$ and $\left(LP - CDOMP^2_{CV}\right)$, respectively. We notice that the feasible region described by $\left(LP - CDOMP^1_{CV}\right)$ is a projection of the feasible region of $\left(LP - CDOMP^2_{CV}\right)$. To see this, choose (x', y', z', t') as a feasible solution of $\left(LP - CDOMP^2_{CV}\right)$. Then (x', y', z', t'') is a feasible solution of $\left(LP - CDOMP^1_{CV}\right)$ by setting $t''_i = \sum_{k=1}^{M} t'_{ik}$ for all $i = 1,\dots,M$. Therefore, following similar arguments to the ones in Section 14.2 in Nickel and Puerto (2005), $\left(CDOMP^1_{CV}\right)$ outperforms $\left(CDOMP^2_{CV}\right)$ in terms of efficiency. Moreover, the following example shows that the inclusion above is strict.

Example 2.1. Let $M = 3$ and $N = 2$, i.e., we have three clients, and we would like to locate two new supply facilities. Let $C = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 3 \\ 3 & 3 & 0 \end{pmatrix}$ be the 3×3 cost matrix. Then, consider the point (x', y', z', t'') defined as follows:

$$x' = \begin{pmatrix} 0 & 0.25 & 0.25 \\ 0.5 & 0 & 0.75 \\ 0.5 & 0.75 & 0 \end{pmatrix}, \quad y' = \begin{pmatrix} 0.5 \\ 0.75 \\ 0.75 \end{pmatrix},$$

$$z' = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{pmatrix}, \quad t'' = \begin{pmatrix} 0.5 \\ 3 \\ 3.5 \end{pmatrix}.$$

This point belongs to the feasible region of $\left(LP - CDOMP^1_{CV}\right)$. To obtain a point belonging to the feasible region of $\left(LP - CDOMP^2_{CV}\right)$ we would need to find values for t'_{ik} for all $i = 1,2,3$. However, this is not possible as on the one hand, by constraints (13), we obtain that

$$t'_{11} \geqslant 0, \quad t'_{12} \geqslant 0.5, \quad t'_{13} \geqslant 0.5,$$

and on the other hand, $\sum_{k=1}^{M} t'_{1k} = 0.5 = t''_1$. Thus, there is no solution to the above system of equations and therefore (x', y', z', t'') cannot be transformed into a solution for $\left(LP - CDOMP^2_{CV}\right)$.

As the formulation $\left(CDOMP^2_{CV}\right)$ is dominated in terms of accuracy by $\left(CDOMP^1_{CV}\right)$, we will only consider the latter formulation in the remainder. Next, we compare formulations $(CDOMP_{CV})$ and $\left(CDOMP^1_{CV}\right)$. Although one may think that formulation $\left(CDOMP^1_{CV}\right)$, which has less variables and constraints, should be more efficient than $(CDOMP_{CV})$ in terms of computational time, our experiences show that this is not the case. In fact, formulation $(CDOMP_{CV})$ outperforms $\left(CDOMP^1_{CV}\right)$, as illustrated in Table 1. This behavior may be expected as a similar conclusion was already obtained for the uncapacitated discrete ordered median problem see Boland et al. (2006). Therefore, in the following, we only consider formulations based on the rationale of the one given in $(CDOMP_{CV})$.

Table 1
Numerical comparison of the two alternative formulations for the $CDOMP_{CV}$.

			Median		Center		k-Centrum		Trimmed mean		∧-Shaped	
	M	N	Avg	Max	Avg	Max	Avg	Max	Avg	Max	Avg	Max
$(CDOMP_{CV})$	6	2	0.1	0.1	1.1	2.3	0.9	2.8	0.2	0.3	0.3	0.4
	7	2	0.2	0.3	11.6	41.5	11.9	29.3	0.9	1.4	0.8	1.0
	8	2	0.6	1.0	57.8	168.9	54.1	175.7	4.5	9.1	3.2	7.8
	9	3	1.2	2.0	2369.1	5586.7	2321.3	5591.8	10.5	18.6	12.3	18.9
	10	3	2.0	2.8	5750.7	7187.4	6041.0	7185.7	203.5	771.1	57.2	116.0
$\left(CDOMP^1_{CV}\right)$	6	2	0.9	1.1	2.2	4.1	2.3	4.2	0.8	1.3	0.9	1.5
	7	2	5.3	8.7	19.9	42.2	20.5	33.5	3.4	4.9	6.2	10.3
	8	2	34.2	82.6	213.6	513.8	115.1	260.8	17.6	30.6	31.2	51.5
	9	3	509.5	1122.8	5453.5	7146.1	4952.6	7142.1	244.2	297.3	481.3	786.3

2.2. Modeling from the supplier point of view

In this section, we address the supplier point of view of a logistics problem. Again, we model it using the capacitated discrete ordered median problem, and thus, we borrow some tools from the literature devoted to the DOMP. This approach leads us to the capacitated discrete ordered median problem from a supplier point of view ($CDOMP_{SV}$). Once more, we are looking for a given number of supply facilities with enough capacity to cover the entire demand of the clients. In contrast to the previous model, we now consider the total transportation costs that accumulate at each supply facility (supplier costs). That is, the supplier costs of a facility are the sum of the transportation costs of shipments from that facility to the clients. The goal is to minimize the weighted sum of the supplier costs to deliver goods for covering the demand of all clients, where the weights are correction factors that depend on the ordered sequence of supplier costs. The interpretation of the correction factors is very similar to the client cost model but now from the point of view of the suppliers.

Note that now the λ vector is in $\mathbb{R}^N$. To formulate this model we use, in addition to the y-variables previously introduced, the following sets of variables:

s_{ijk} = proportion of the capacity of supplier j sent to customer k when the transportation costs of supplier j are the ith smallest value of the supplier cost vector

$$z_{ij} = \begin{cases} 1 & \text{if the costs of supplier } j \text{ are in position } i, \\ 0 & \text{otherwise,} \end{cases}$$

for $i = 1,\dots,N$ and $j,k = 1,\dots,M$. Then, the model, denoted ($CDOMP_{SV}$), is defined as

$$\min \quad \sum_{i=1}^{N} \lambda_i \sum_{j=1}^{M} \sum_{k=1}^{M} s_{ijk} b_j c_{jk} \tag{16}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} \sum_{j=1}^{M} s_{ijk} b_j = a_k \quad \forall k = 1,\dots,M \tag{17}$$

$$\sum_{k=1}^{M} s_{ijk} \leqslant z_{ij} \quad \forall i = 1,\dots,N, j = 1,\dots,M \tag{18}$$

$$\sum_{j=1}^{M} z_{ij} \leqslant 1 \quad \forall i = 1,\dots,N \tag{19}$$

$$\sum_{i=1}^{N} z_{ij} \leqslant y_j \quad \forall j = 1,\dots,M \tag{20}$$

$$\sum_{j=1}^{M} \sum_{k=1}^{M} s_{ijk} b_j c_{jk} \leqslant \sum_{j=1}^{M} \sum_{k=1}^{M} s_{i+1,jk} b_j c_{jk} \quad \forall i = 1,\dots,N-1 \tag{21}$$

$$\sum_{j=1}^{M} y_j = N$$

$$s_{ijk} \geqslant 0 \quad \forall i = 1,\dots,N, j,k = 1,\dots,M$$

$$y_j \in \{0,1\} \quad \forall j = 1,\dots,M$$

$$z_{ij} \in \{0,1\} \quad \forall i = 1,\dots,N, j = 1,\dots,M$$

The objective function (16) represents the weighted ordered sum of total supplier costs (the supplier costs in position i are $\sum_{j=1}^{M}\sum_{k=1}^{M} s_{ijk} b_j c_{jk}$). Constraints (17) guarantee that the demand of all clients is covered. Constraints (18)–(20) ensure that total transportation costs that accumulate at a supply facility may be allocated to at most one position in the vector of supplier costs. Constraints (21) guarantee the non-decreasing order of the entries in the cost vector.

Comparing the modeling meaning of the formulation $CDOMP_{CV}$ with $CDOMP_{SV}$, the main difference appears on the ordering constraints (5) (in $CDOMP_{CV}$) with respect to (21) (in $CDOMP_{SV}$). The former aggregates cost on clients whereas the latter does on suppliers, each model captures the intrinsic importance of the driving force in the logistic system.

2.3. Modeling from the logistics provider point of view

In this section, we discuss the logistics provider point of view of a logistics problem. We model this problem via the capacitated discrete ordered median problem from a logistics provider point of view ($CDOMP_{LV}$). The logistics provider wants to study the overall costs of a certain solution by looking at all transportation links separately. The goal is to minimize the weighted sum of the transportation costs from each supplier to each client. The weights are correction factors that depend on the ordered sequence of the costs of the single transportation links. The interpretation of the correction factors is very similar to the one given for the client cost model but in this case we consider the transportation costs from each server to each client.

To formulate the model we need, in addition to the y-variables, the following two sets of variables:

s_{ijk} = proportion of demand a_k covered by supplier j when the transportation costs of that proportion of demand are the ith lowest value of the transportation cost vector between suppliers and customers.

$$\delta_{ijk} = \begin{cases} 1 & \text{if the costs for supplying client } k \text{ from supplier } j \text{ are in position } i, \\ 0 & \text{otherwise,} \end{cases}$$

for $i = 1,\dots,NM$ and $j, k = 1,\dots,M$. Now, the formulation of this model, denoted ($CDOMP_{LV}$), is the following:

$$\min \quad \sum_{i=1}^{NM}\lambda_i\sum_{j=1}^{M}\sum_{k=1}^{M}s_{ijk}\,a_k\,c_{jk} \tag{22}$$

$$\text{s.t.} \quad \sum_{i=1}^{NM}\sum_{j=1}^{M}s_{ijk} = 1 \quad \forall k=1,\dots,M \tag{23}$$

$$s_{ijk} \leqslant \delta_{ijk} \quad i=1,\dots,NM,\ j,k=1,\dots,M \tag{24}$$

$$\sum_{j=1}^{M}\sum_{k=1}^{M}\delta_{ijk} \leqslant 1 \quad i=1,\dots,NM \tag{25}$$

$$\sum_{i=1}^{NM}\delta_{ijk} \leqslant 1 \quad \forall j,k=1,\dots,M \tag{26}$$

$$\sum_{j=1}^{M}\sum_{k=1}^{M}s_{ijk}\,a_k\,c_{jk} \leqslant \sum_{j=1}^{M}\sum_{k=1}^{M}s_{i+1,jk}\,a_k\,c_{jk} \quad \forall i=1,\dots,NM-1 \tag{27}$$

$$\sum_{i=1}^{NM}\sum_{k=1}^{M}s_{ijk}\,a_k \leqslant b_j\,y_j \quad \forall j=1,\dots,M \tag{28}$$

$$\sum_{j=1}^{M}y_j = N$$

$$y_j \in \{0,1\} \quad \forall j=1,\dots,M$$

$$s_{ijk} \geqslant 0 \quad \forall i=1,\dots,NM,\ \forall j,k=1,\dots,M$$

$$\delta_{ijk} \in \{0,1\} \quad i=1,\dots,NM,\ j,k=1,\dots,M$$

The objective function (22) represents the weighted ordered sum of the transportation costs from each supplier to each client (the transportation costs from a supplier to a client in the ith position are $\sum_{j=1}^{M}\sum_{k=1}^{M}s_{ijk}a_kc_{jk}$). Constraints (23) guarantee that the demand of all clients is covered. Constraints (24) and (25), together with the integrality constraints on the δ_{ijk}-variables, ensure that we can assign to a position i the transportation costs of at most one transportation link. In turn, constraints (26) guarantee that the costs of supplying a client from a supplier can appear in at most one position in the cost vector. Constraints (27) ensure the non-decreasing order of the entries of the cost vector. Constraints (28) guarantee that the total amount supplied by each server does not exceed its capacity.

Once again the main difference of this model with respect to the client and supplier models is on the elements to be ordered. Whereas on the previous models the driving forces are the clients or suppliers here one cares on transportation links. Hence, these are the amounts that are aggregated and compensated via λ-weights.

3. Improved formulations

The formulations presented in the previous section does not make any hypothesis on the sequence of the λ modeling weights. Thus, although providing great generality, these formulations are at times not very efficient. One way to improve the efficiency is by imposing conditions on the feasible λ weights. This has been already done by Ogryczak and Tamir (2003) and Ogryczak and Śliwiński (2003) who require non-decreasing monotonicity on the sequence of λ. The reader may notice that even though this is a restriction, most of the classical models in locational analysis still fall within this class. We have adapted the formulation by Ogryczak and Tamir (2003) and Ogryczak and Śliwiński (2003) for the ordered median problem to our case. Basically, given a set of variables z_i, they give a linear programming formulation to compute the q largest z-values as follows. $z_{(i)}$ represents the values z_i sorted in non increasing order.

For any integer $q, 1 \leqslant q \leqslant M$, consider the following function defined in $[0,+\infty)$:

$$f_q(t) := qt + \sum_{i=1}^{M}\max\{0, z_i - t\}.$$

This is a convex piecewise linear function with slopes moving from $q-M$ to q in integer steps, whose minimum is reached either when the slope is 0 or, if this is not the case, when the slope changes from negative to positive, i.e., when t equals the qth maximum value of the vector z, namely $z_{(q)}$. Thus, the minimum value of f is

$$f_q(z_{(q)}) = qz_{(q)} + \sum_{i=1}^{M}\max\{0, z_i - z_{(q)}\} = qz_{(q)} + \sum_{i=1}^{q}(z_{(i)} - z_{(q)}) = \sum_{i=1}^{q}z_{(i)},$$

i.e., the sum of the q maximum values. Therefore, minimizing the sum of the q-largest z-values can be linearized as

$$\min\ qt_q + \sum_{i=1}^{M}d_{iq} \quad \text{s.t.} \quad d_{iq} \geqslant 0\ \forall i, \quad d_{iq} \geqslant z_i - t_q\ \forall i. \tag{29}$$

Depending on the different points of view, non-decreasing monotonicity on the sequence of λ means that: (1) $0 \leqslant \lambda_1 \leqslant \cdots \leqslant \lambda_M$ for the CDOMP$_{CV}$, denoted in the following by CDOMP$^{\leqslant}_{CV}$, (2) $0 \leqslant \lambda_1 \leqslant \cdots \leqslant \lambda_N$ for the CDOMP$_{SV}$, that we denote by CDOMP$^{\leqslant}_{SV}$, and (3) $0 \leqslant \lambda_1 \leqslant \cdots \leqslant \lambda_{MN}$ for the CDOMP$_{LV}$ denoted by CDOMP$^{\leqslant}_{LV}$. The formulations proposed in this section are based on the above transformation. Following the scheme of the previous sections, we propose different formulations that depend on the different points of view considered in the planning process. We will set $\lambda_0 = 0$ to simplify the notation.

3.1. Client point of view

To state the improved model for the client point of view, CDOMP$^{\leqslant}_{CV}$, we first define the following variables:

s_{jk} = proportion of the demand of a_k satisfied by supplier j

for $j, k = 1,\dots,M$. Then, the formulation, denoted $(CDOMP^{\leqslant}_{CV})$, is as follows:

$$\min \quad \sum_{i=1}^{M}(\lambda_{M-i+1} - \lambda_{M-i})\left(i \cdot t_i + \sum_{k=1}^{M}d_{ki}\right) \tag{30}$$

$$\text{s.t.} \quad d_{ki} \geqslant a_k\sum_{j=1}^{M}s_{jk}\,c_{jk} - t_i \quad \forall i,k=1,\dots,M \tag{31}$$

$$\sum_{j=1}^{M}s_{jk} = 1 \quad \forall k=1,\dots,M \tag{32}$$

$$\sum_{k=1}^{M}s_{jk}\,a_k \leqslant b_j\,y_j \quad \forall j=1,\dots,M$$

$$\sum_{j=1}^{M}y_j = N$$

$$y_j \in \{0,1\} \quad \forall j=1,\dots,M$$

$$s_{jk} \geqslant 0 \quad \forall j,k=1,\dots,M$$

$$d_{ki} \geqslant 0 \quad \forall i,k=1,\dots,M$$

$$t_i \in \mathbb{R} \quad \forall i=1,\dots,M$$

The objective function (30) and the constraints (31) represent the ordered weighted sum of the values $a_k\sum_{j=1}^{M}s_{jk}c_{jk}$ for $k=1,\dots,M$, as shown in (29) (see Ogryczak and Tamir (2003) for details). Constraints (32) guarantee that for a fixed $k=1,\dots,M$ the variables s_{jk} represent a proportion when the index j takes the values $1,\dots,M$. The remaining constraints are the usual ones for capacitated location problems.

3.2. Supplier point of view

To give the improved model for the supplier point of view, $\text{CDOMP}^{\leqslant}_{SV}$, we first define the following variables:

s_{jk} = proportion of the capacity of supplier j sent to customer k

for $j, k = 1, \dots, M$. Then, the formulation, denoted $(CDOMP^{\leqslant}_{SV})$, is the following:

$$\min \quad \sum_{i=1}^{N} (\lambda_{N-i+1} - \lambda_{N-i}) \left(i \cdot t_i + \sum_{j=1}^{N} d_{ji} \right) \tag{33}$$

$$\begin{aligned} \text{s.t.} \quad & d_{ji} \geqslant \sum_{k=1}^{M} s_{jk} b_j c_{jk} - t_i \quad \forall i = 1, \dots, N, j = 1, \dots, M \qquad (34) \\ & \sum_{j=1}^{N} s_{jk} b_j = a_k \quad \forall k = 1, \dots, M, \\ & \sum_{k=1}^{N} s_{jk} \leqslant y_j \quad \forall j = 1, \dots, M \\ & \sum_{j=1}^{M} y_j = N \\ & y_j \in \{0,1\} \quad \forall j = 1, \dots, M \\ & s_{jk} \geqslant 0 \quad \forall j = 1, \dots, M, k = 1, \dots, M \\ & d_{ki} \geqslant 0 \quad \forall i = 1, \dots, N, k = 1, \dots, M \\ & t_i \in \mathbb{R} \quad \forall i = 1, \dots, N \end{aligned}$$

The objective function (33) and the constraints (34) model the ordered weighted sum of the values $\sum_{k=1}^{M} s_{jk} b_j c_{jk}$ for $j = 1, \dots, M$, as shown in (29) (see Ogryczak and Tamir (2003) for further details). The remaining constraints are the usual ones for capacitated location problems.

3.2.1. Logistics provider point of view

To give the improved model for the logistic provider point of view, $\text{CDOMP}^{\leqslant}_{LV}$, we first define the following variables:

s_{jk} = proportion of the demand of a_k covered by the supplier j

for $j, k = 1, \dots, M$. Then, the formulation, denoted $(CDOMP^{\leqslant}_{LV})$, is the following:

$$\min \quad \sum_{i=1}^{NM} (\lambda_{NM-i+1} - \lambda_{NM-i}) \left(i \cdot t_i + \sum_{j=1}^{M} \sum_{k=1}^{M} d_{jki} \right) \tag{35}$$

$$\begin{aligned} \text{s.t.} \quad & d_{jki} \geqslant a_k s_{jk} c_{jk} - t_i \quad \forall j, k = 1, \dots, M, i = 1, \dots, NM \qquad (36) \\ & \sum_{j=1}^{M} s_{jk} = 1 \quad \forall k = 1, \dots, M \\ & \sum_{k=1}^{M} s_{jk} a_k \leqslant b_j y_j \quad \forall j = 1, \dots, M \\ & \sum_{j=1}^{M} y_j = N \\ & y_j \in \{0,1\} \quad \forall j = 1, \dots, M \\ & s_{jk} \geqslant 0 \quad \forall j, k = 1, \dots, M \\ & d_{jki} \geqslant 0 \quad \forall j, k = 1, \dots, M, i = 1, \dots, NM \\ & t_i \in \mathbb{R} \quad \forall i = 1, \dots, NM \end{aligned}$$

The objective function (35) and the constraints (36) model the ordered weighted sum of the values $a_k s_{jk} c_{jk}$ for $j, k = 1, \dots, M$, as shown in (29) (see Ogryczak and Tamir (2003) for further details). The remaining constraints are the usual ones for capacitated location problems.

4. Computational results

In this section we present computational results for the proposed models to give an idea about the capabilities of the different formulations. For the tests we randomly generated problem instances of varying size. Here, we chose the following parameters:

- Number of clients: Depending on the model, the values for M range from 6 to 70.
- Location of the clients: The coordinates of the clients are uniformly distributed in the square $[0,10] \times [0,10]$.
- Demand of clients: The demands are uniformly distributed in the interval [10,20].
- Transportation cost: The costs are computed as $c_{ij} = r_i d(i,j)$ where r_i is a uniform random variable in the interval [1,5] (modeling the transportation cost rate) and $d(\cdot,\cdot)$ is the Euclidean distance.
- Number of suppliers: N is chosen proportional to the number of clients: $N = \lceil M/b \rceil$, where $b = 4$ for the general formulations and $b = 5$ for the alternative formulations introduced in Section 3. (b is the average number of clients per facility.)
- Capacity of suppliers: The capacities are uniformly distributed in the interval $\left[1.1 \frac{\sum_{i=1}^{M} a_i}{N}, 1.3 \frac{\sum_{i=1}^{M} a_i}{N}\right]$.
- Modeling vector λ: We consider five different types of vectors
  - L1(*Median*): λ corresponding to the median problem, i.e., $\lambda = (1, \dots, 1)$.
  - L2 (*Center*): λ corresponding to the center problem, i.e., $\lambda = (0, \dots, 0, 1)$.
  - L3 (*kCentrum*): λ corresponding to the k-centrum model, i.e., $\lambda = (0, \dots, 0, 1, \overset{k}{\dots}, 1)$ where $k = \lceil 0.4M \rceil$.
  - L4 (*TrimmedMean*): λ corresponding to the (k_1, k_2)-trimmed mean problem, i.e., $\lambda = (0, \overset{k_1}{\dots}, 0, 1, \dots, 1, 0, \overset{k_2}{\dots}, 0)$ where $k_1 = k_2 = \lceil 0.2M \rceil$.
  - L5 ($\wedge$-*shaped*): λ corresponding to the $\wedge$-shaped problem with values $(0, 2, 4, \dots, M-3, M-1, M-2, M-4, \dots, 3, 1)$ if M is odd, and $(0, 2, 4, \dots, M-2, M-1, M-3, \dots, 3, 1)$ otherwise.

For each number of clients M we randomly generated 10 instances. We solved the instances using CPLEX 9.1 where we limit the maximal running time to 2 hours. All tests were done on a PC with an Intel Pentium 4 processor with 3.4 GHz and 2 GB Ram. In what follows, we report the average and maximum running times in seconds for each model. For the client point of view, we introduced three models: $(CDOMP_{CV})$, $\left(CDOMP^2_{CV}\right)$, and $\left(CDOMP^1_{CV}\right)$. However, as the second formulation is inferior to the third (see Section 2.1.1), we present in Table 1 results just for the first and third formulation. In the table, 'M' and 'N' denote the number of clients and new facilities, respectively, and 'Avg' and 'Max' the average and, respectively, maximal running times in seconds over all instances of a given size.

Considering the first formulation, $(CDOMP_{CV})$, we observe that the choice of the modeling vector has a great impact on the running times. Whereas the problems for the L1, L4, and L5 modeling vector (*Median, TrimmedMean*, and $\wedge$-*shaped*) can be solved within a couple of minutes, the other two λ-vectors result in running times of well over 1 hour already for 10 clients and three new facilities. For the center (k-centrum) objective we could solve just 3 out of 10 (2 out of 10) instances optimally within 2 hours for $M = 10$.

Comparing the formulations $(CDOMP_{CV})$ and $\left(CDOMP^1_{CV}\right)$, we observe that although the number of variables and constraints in the latter formulation is much smaller compared to former, the computational tests prove the formulation $(CDOMP_{CV})$ to be more efficient for all five modeling vectors. For $\left(CDOMP^1_{CV}\right)$, already for

Table 2
Results for the supplier and logistics provider point of view ($CDOMP_{SV}$ and $CDOMP_{LV}$).

			Median		Center		k-Centrum		Trimmed mean		∧-Shaped	
	M	N	Avg	Max	Avg	Max	Avg	Max	Avg	Max	Avg	Max
Supplier	16	4	0.7	1.6	3.8	10.1	3.8	13.1	1.2	1.5	1.2	2.0
	18	5	2.7	3.6	66.4	150.1	66.0	147.2	6.2	16.5	8.8	12.1
	20	5	3.5	5.7	146.5	501.7	135.4	394.9	9.5	18.9	12.1	18.2
	22	6	11.5	30.7	1614.4	3512.6	1844.6	3594.6	120.8	307.7	94.3	139.7
	24	6	14.2	44.6	3154.9	7003.8	3118.8	7057.0	164.0	846.3	139.8	440.2
	26	7	191.2	976.2	6335.2	7153.0	6367.0	7184.0	491.0	2166.4	785.2	2100.5
Log	6	2	2.1	4.1	7100.4	7128.5	7083.3	7152.4	1069.3	2305.7	3590.5	7185.0

nine clients CPLEX often reached the 2 hour time limit without finding the optimal solution (this happened for four instances for the center and two instances for the *k*-centrum objective). Therefore, we consider in the following only the formulation ($CDOMP_{CV}$).

In Table 2, we give results for the supplier and logistics provider point of view.

Again we observe that, as for the client point of view, the running times for the center and *k*-centrum problem are much higher than for the other three problem types. (Note that for the supplier point of view we could solve for $M = 26$ just 6 out of 10 (7 out of 10) instances for the center (*k*-centrum) objective optimally within 2 hours; for the logistics provider point of view these numbers increase to 9 and 8, respectively, out of 10.) If we now compare the results for the client, supplier, and logistics provider models, we see that the approaches are not just different from a modeling point of view, but also from the computational side. The $CDOMP_{LV}$ is by far the most difficult model. For the center and *k*-centrum objective, already for the smallest problem size with six clients almost none of the instances could be solved to optimality (within 2 hours). In contrast to that, for the supplier point of view we can solve instances to optimality that are more than twice as large compared to $CDOMP_{CV}$.

4.1. Improved formulations

In Section 3 we introduced alternative formulations for the CDOMP for modeling vectors λ that consist of non-decreasing values. The following results show that these models are much more efficient than the general ones, considerably increasing the size of the instances which can be solved to optimality (within 2 hours).

In Table 3 we give the results for the improved formulations for all three different points of view. We observe that the size of the problems we can solve to optimality for the client and logistics cost model is 5–6 times larger than for the general model. The supplier cost model however shows a completely different behavior. First, the tractable problem size increases only slightly. Moreover, for instances with 40 clients, CPLEX failed to obtain a solution in all cases due to insufficient memory (the branch and bound tree was larger than 2 GB). And last, in contrast to previous observations, the median problem is on average more difficult than the center and *k*-centrum problem.

5. Conclusions and outlook

The great importance of discrete location problems in strategic supply chain planning has stipulated the need to develop new location models that fit better to real situations. Taking advantage of a family of flexible models in Location Theory introduced by Nickel and Puerto (2005) and Boland et al. (2006), this paper provides new formulations of capacitated discrete location problems within this framework. These new formulations are beyond of simply academic extensions of classical location models since they allow to incorporate some actual factors that have been neglected up-todate: the different points of view of the parties in the logistic system.

This paper covers the above aspects by introducing new formulations of capacitated discrete location problems. Although, we focus mainly on modeling issues, we also report computational tests comparing the performance of the different formulations. These results aim to establish the limits, both in CPU-time and size, of the exact resolution of the different models using standard MIP solvers. Our preliminary analysis shows that "ad-hoc" methods are required to solve even small sized instances of these problems. Thus, this paper provides a starting point for the development of exact and heuristic solution methods for all the models that have been introduced. The works by Domínguez-Marín et al. (2005) (heuristic) and Marín et al. (2008) (exact) for the uncapacitated problem can serve as a basis for these investigations.

This paper, for the sake of brevity, does not consider models with setup costs, where a similar analysis of the different points

Table 3
Running times for all three points of view for the improved formulations of the CDOMP.

View			Median		Center		k-Centrum	
	M	N	Avg	Max	Avg	Max	Avg	Max
Client	30	6	0.2	0.7	8.4	18.9	5.8	13.1
	40	8	0.5	1.6	235.9	619.2	55.8	138.3
	50	10	1.6	4.9	1364.4	4555.0	1217.5	3898.5
	60	12	1.2	1.5	6294.0	7197.8	4987.8	7145.1
Supplier	20	4	8.5	9.5	7.3	9.3	7.5	9.2
	25	5	83.7	111.0	54.3	84.0	57.3	104.9
	30	6	510.4	635.9	255.3	380.0	249.7	366.7
	35	7	4376.1	4968.1	3153.8	5404.5	3207.1	5802.3
Logistics	10	3	0.2	0.2	0.3	0.3	0.3	0.3
	20	4	3.9	5.3	42.9	52.9	46.7	61.2
	30	6	26.5	32.0	7140.4	7158.9	7148.7	7163.3

of view of the logistic problem is possible. The interested reader is referred to Kalcsics et al. (2009) for an extended analysis of this type of models under the framework of the Capacitated Ordered Median Location Problem. Also models with an integrated objective that considers two or more perspectives or a multi-criteria formulation would be interesting to study.

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